Mendeleev Commun., 2004, 14(1), 22–23

Influence of a liquid flow through a foam under a pressure drop on the Plateau border curvature profile

Natalya G. Vilkova and Pyotr M. Kruglyakov

Penza State University of Architecture and Building, 440028 Penza, Russian Federation. Fax: + 7 8412 43 7277; e-mail: kpyotr10@gasa.penza.com.ru

DOI: 10.1070/MC2004v014n01ABEH001769

The experimental Plateau border curvature profiles are compared to the profiles calculated on the assumption of a mobile border surface.

The dispersion medium flow out from foams and emulsions is a main stage of their destruction. In the hydrodynamics of dispersed systems with elastic surfaces, the mobility of these surfaces, which influence greatly the total flow velocity and the Plateau border profile (a change in the Plateau border curvature with height), is very important.

The aim of this work was to study the Plateau border profile in foams of sodium dodecylsulfate (SDS) and Triton X-100 with an additional compound (electrolyte, glycerol, or gelatine) and a comparison of the experimental Plateau border profile and a calculated one assuming that border surfaces are mobile. The experimental cell was described earlier.[1,2]

Foam from a foam generator was introduced into a cell *1* with two porous plates *3*,*4* which were in contact with a liquid (dispersion medium) under different pressure drops $P_L^{max} = P_0 - \Delta P_{min}$ and $P_L^{min} = P_0 - \Delta P_{max}$ at the upper and lower porous plates, respectively (Figure 1).

On the assumption that the border surfaces are immobile and $dP_L/dl > \rho g$ [where P_L is the liquid pressure in the Plateau border; ρ is the liquid density; g is the acceleration due to gravity; and l is the coordinate in the direction opposite to the liquid flow (Figure 1)], the theoretical volumetric flow rate (Q) is determined by the Leonard–Lemlich equation:[2]

$$Q = 0.16\frac{f}{\eta}\left(\frac{dP}{dl}\right)r^4, \qquad (1)$$

where Q is the volumetric flow rate; r is the Plateau border radius; f is a geometrical dimensionless coefficient of 3.3×10^{-3};[2] η is the dynamic viscosity of the solution; and dP/dl is the pressure gradient in the liquid phase.

The functions $r(l)$ were calculated by the equation:[1,2]

$$r^3 = r_{min}^3 + \frac{(r_{max}^3 - r_{min}^3)l}{L}, \qquad (2)$$

where l is the distance from the cell bottom; $L = 1.35H$; H is the straight-line distance from one porous plate to the other; r_{min} and r_{max} are minimum and maximum Plateau border radii at the lower and upper porous plates, which were under the different pressure drops P_L^{max} and P_L^{min}, respectively; 1.35 is the winding coefficient for a foam bubble as a pentagonal dodecahedron.

The Plateau border radius was calculated by the equation

$$r = \sigma/P_\sigma, \qquad (3)$$

where σ is the surface tension and P_σ is the capillary pressure (the difference between the pressure in the foam bubbles P_b and the pressure in the liquid phase P_L).

In the polyhedral foams with the volume part of a liquid phase approximately equal to 5×10^{-3}, the excess pressure in the liquid phase ($\Delta P_L = P_L - P_0$) is measured by a capillary micromanometer,[2] and it is much higher than the excess bubble gas pressure $\Delta P_b = P_0 - P_b$; therefore, $P_\sigma \approx -\Delta P_L$ (here, P_0 is the pressure in a surrounding medium).

For the estimation of the border surface mobility, Desai and Kumar[3–5] used a foam bubble model of a pentagonal dodecahedron, whose border shape is ideally considered as a pipe with a cross section as an equilateral triangle, and its vertex has a zero velocity. The Navier–Stoke equation was solved for this model using the method of successive approximations involving surface viscosity and assuming that border edges are immobile (at the border-film transition line). The border surface mobility was estimated by the parameter

$$\beta = Q_1/Q_{th} = f(\alpha), \qquad (3')$$

where Q_1 is the volumetric flow velocity through the border with a mobile surface; Q_{th} is the theoretical flow velocity through a border with an immobile surface; α is the inverse value of surface viscosity ($\alpha = 0.176r\eta/\eta_s$), where η_s is the surface viscosity. The function $\beta(\alpha)$ was described elsewhere.[3–5]

The function $\beta = f(\alpha)$ was linearised in the $\lg\beta$–$\lg\alpha$ coordinates.

With the help of this straight line, we obtained the approximate formula:

$$\beta = 1 + 5.4\alpha^{0.5}. \qquad (4)$$

From equations (1), (3′) and (4) we obtained equation (5) for calculating the volumetric flow rate of the solution taking into consideration the surface mobility

$$Q_1 = 0.16\frac{f}{\eta}\left(\frac{dP}{dl}\right)r^4(1 + 5.4\alpha^{0.5}). \qquad (5)$$

Substituting (3) in (5) and then separating the variables and integrating the left-hand side of this equation from $l = 0$ to $l = 1$ and the right-hand side from $r = r_{min}$ to $r = r$, we obtained the equation

$$\frac{Ql}{A\sigma} = \frac{r^3}{3} - \frac{r_{min}^3}{3} + 0.66\left(\frac{\eta}{\eta_s}\right)^{0.5}(r^{3.5} - r_{min}^{3.5}), \qquad (6)$$

where $A = 0.16f/\eta$.

The volumetric flow rate (Q'_{th}) of the solution through the Plateau border with definite r_{min} and r_{max} is

$$Q'_{th} = \frac{A\sigma}{L}\left[\frac{r_{max}^3}{3} - \frac{r_{min}^3}{3} + 0.66\left(\frac{\eta}{\eta_s}\right)^{0.5}(r_{max}^{3.5} - r_{min}^{3.5})\right]. \qquad (7)$$

From equations (6) and (7), we obtain the following equation for calculating the Plateau border curvature profile:

$$\frac{1}{L}\left[r_{max}^3 - r_{min}^3 + 3\cdot1.98\left(\frac{\eta}{\eta_s}\right)^{0.5}(r_{max}^{3.5} - r_{min}^{3.5})\right] + r_{min}^3 + 1.98r_{min}^{3.5}\left(\frac{\eta}{\eta_s}\right)^{0.5} = r^3 + 1.98r^{3.5}\left(\frac{\eta}{\eta_s}\right)^{0.5}. \qquad (8)$$

Using equation (8), we calculated the Plateau border profile of foams of SDS with an electrolyte and gelatine added and of Triton X-100 (Table 1, r_2). As can be seen in Table 1, the Plateau border radii were greater than the radii (r_1) calculated by equation (2) on the assumption of rigid border surfaces.

The problem of a liquid flow in a single Plateau border and the influence of the interfacial surface viscosity on the liquid velocity was investigated by Nguen.[6] The volumetric flow rate of a surfactant solution in the Plateau border was calculated as

$$Q_2 = \frac{Kr^4}{\eta}\frac{dP}{dl}\left[\frac{a(B_0)^{-1/2}}{C + (B_0)^{0.628}} + 0.02\right], \qquad (9)$$

where $C = 0.209$; $K = 0.026$; $a = 0.0655$; B_0 is the Boussinesq number defined as the ratio of the surface to the bulk dynamic viscosity:

Mendeleev Commun., 2004, 14(1), 22–23

$$B_0 = \frac{\eta_s}{\eta r};\; B_0 = \frac{0.176}{\alpha}. \qquad (10)$$

Note that for all of the test surfactant solutions the value $B_0^{0.628} >> C$ (for example, the lowest interfacial viscosity $\eta_s = 1.2\times10^{-7}$ n s m^{-1} and, for the greatest Plateau border radius, $r = 32\times10^{-6}$ m, $B_0 = 2.3$).

Substituting (3) into (9), separating variables and integrating the left-hand side of this equation from 0 to 1 and the right-hand side from r_{min} to r, we obtain:

$$Q_2 = \frac{K\sigma a\sqrt{\frac{\eta}{\eta_s}}\,(r^{4.13} - r_{min}^{4.13})l}{\eta\left(\frac{\eta}{\eta_s}\right)^{0.63}4.13L} + \frac{5.2\times10^{-4}\sigma(r^3 - r_{min}^3)l}{3\eta L}. \qquad (11)$$

Taking into account the volumetric flow rate (Q') of the solution through the Plateau border with minimum r_{min} and maximum r_{max} radii

$$Q' = \frac{K\sigma a\sqrt{\frac{\eta}{\eta_s}}\,(r_{max}^{4.13} - r_{min}^{4.13})l}{\eta\left(\frac{\eta}{\eta_s}\right)^{0.63}4.13L} + \frac{5.2\times10^{-4}\sigma(r_{max}^3 - r_{min}^3)l}{3\eta L}. \qquad (12')$$

we determined the Plateau border profile:

$$\frac{K\sigma a(\eta/\eta_s)^{0.5}r^{4.13}}{4.13\eta(\eta_s/\eta)^{0.63}} + \frac{5.2\times10^{-4}\sigma r^3}{3\eta} = \frac{K\sigma a(\eta/\eta_s)^{0.5}(r_{max}^{4.13} - r_{min}^{4.13})l}{4.13\eta(\eta_s/\eta)^{0.63}L} +$$
$$\frac{5.2\times10^{-4}\sigma(r_{max}^3 - r_{min}^3)l}{3\eta L} + \frac{K\sigma a(\eta/\eta_s)^{0.5}r_{min}^{4.13}}{4.13\eta(\eta_s/\eta)^{0.63}} + \frac{5.2\times10^{-4}\sigma r_{min}^3}{3\eta} \qquad (12)$$

Using equation (12), we calculated the Plateau border radii (r_3) in the foams of SDS and Triton X-100 (Table 1).

We found that in the foam of SDS with common black films and in the foam of Triton X-100, the calculated [by equation (12)] radii are greater by 0.5–1% (in the middle part of the Plateau border) than the Plateau border radii on the assumption that border 'walls' are immobile [equation (2)] (at $\Delta P_{max} = 8$ kPa; $\Delta P_{min} = 3$ kPa and $r_{min} = 4\times10^{-6}$ m).

The experimental Plateau radii in the foams of SDS + 0.1 M NaCl and Triton X-100 + 0.4 M NaCl were the same as calculated by equation (12) at the high pressure gradients ($\Delta P_{max} = 8$ kPa; $\Delta P_{min} = 3$ kPa) and equal to 8×10^{-6} and 8.6×10^{-6} m, respectively.

The Plateau border profile was also studied at the pressure gradients $\Delta P = 3$ kPa ($\Delta P_{max} = 4$ kPa; $\Delta P_{min} = 1$ kPa) and the maximum radius 30×10^{-6} m in the foams of SDS and Triton X-100 with additional compounds (electrolyte, gelatine and glycerol).

The Plateau border profiles in the foam of SDS with an additional compound (electrolyte and gelatine) are given in Figure 2.

As can be seen in Figure 2, the experimental Plateau border radius in the foam of SDS + 0.1 M NaCl was equal to 24×10^{-6} m (at $l = 1$ cm), and the Plateau border radius calculated from equation (12) was 24.1×10^{-6} m. The experimental Plateau border radius in the foam of Triton X-100 + 0.4 M NaCl and the calculated value were 24×10^{-6} and 26×10^{-6} m, respectively (Table 1).

Table 1 The Plateau border radii[a] in forms of SDS and Triton X-100.

Test surfactant solution	ΔP_{max}/kPa	ΔP_{min}/kPa	L/m	r_{exp}/10^{-6} m	r_1/10^{-6} m	r_2/10^{-6} m	r_3/10^{-6} m
10^{-3} M SDS + 0.4 M NaCl	8	3	0.02	6.6	8.64	8.65	8.71
10^{-3} M SDS + 0.1 M NaCl	8	3	0.02	8	8.07	8.1	8.1
10^{-3} M Triton X-100 + 0.4 M NaCl	8	3	0.02	8.6	8.64	8.7	8.73
10^{-2} M SDS + 0.1 M NaCl + 0.2% gelatine	4	1	0.02	17	23.9	23.9	23.9
10^{-3} M Triton X-100 + 0.4 M NaCl	4	1	0.02	24	25.5	26	26.5
10^{-3} M SDS + 0.1 M NaCl	4	1	0.02	24	23.9	24.1	24

[a]The Plateau border radii are given at $l = 1$ cm.

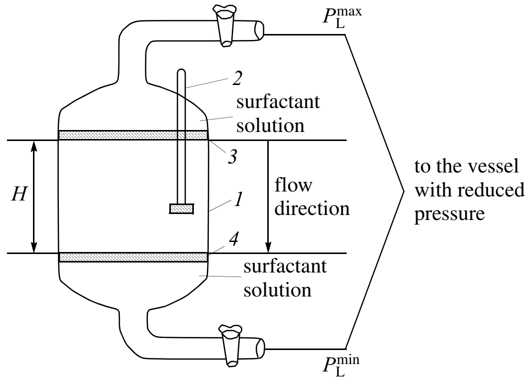

Figure 1 Schematic diagram of the cell for investigation of foams: (*1*) cell; (*2*) micromanometer; (*3*),(*4*) - porous plates; *H* is the foam height.

(Note that the Plateau border radii in the foam of Triton X-100 + 0.4 M NaCl and glycerol were the same as in Table 1).

In the foam of SDS with Newton black films and gelatine, the Plateau radii were distinguished from those calculated by equation (12).

In the foam of SDS with Newton black films, the Plateau border radius was 20% smaller than that calculated by equation (2) with the surface immobility taken into account and equal to 6.6×10^{-6} m ($\Delta P_{max} = 8$ kPa; $\Delta P_{min} = 3$ kPa and $l = 1$ cm).

A similar decrease in the Plateau border radii was observed in the foam of SDS and 0.2% gelatine. The experimental Plateau border radius in the foam of SDS + 0.1 M NaCl + 0.2% gelatine was 17×10^{-6} m (at $l = 1$ cm), which differed by 28% from that calculated by equation (2).

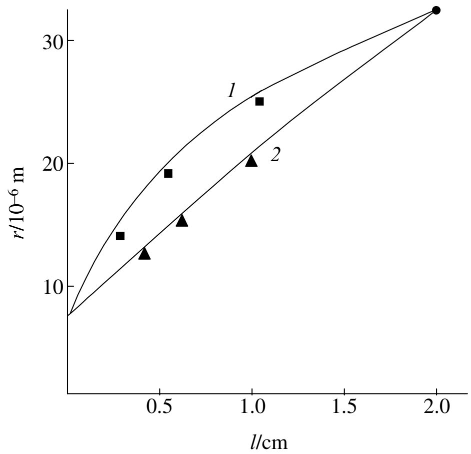

Figure 2 The function $r(l)$ in the foam of 10^{-3} mol dm^{-3} SDS: $\Delta P_{max} = 4$ kPa; $\Delta P_{min} = 1$ kPa: (*1*) calculation by equation (12); (■) the experimental radii in the foam of SDS + 0.1 M NaCl; (*2*) (▲) the experimental radii in the foam of SDS + 0.1 M NaCl + gelatine.

References

1 N. G. Fokina and P. M. Kruglyakov, *Kolloidn. Zh.*, 1986, **40**, 318 [*Colloid J. USSR* (*Engl. Transl.*), 1986, **40**, 268].

2 P. M. Kruglyakov and D. Exerowa, *Foam and Foam Films*, Elsevier, Amsterdam, 1998.

3 D. Desai and R. Kumar, *Chem. Eng. Sci.*, 1982, **37**, 1361.

4 D. Desai and R. Kumar, *Chem. Eng. Sci.*, 1983, **38**, 1525.

5 D. Desai and R. Kumar, *Chem. Eng. Sci.*, 1984, **39**, 1559.

6 A. Nguyen, *J. Colloid Interface Sci.*, 2002, **249**, 194.

Received: 21st April 2003; Com. 03/2095